Energy-Turn-Rate Characteristics and Turn Performance of an Aircraft

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Specific energy of turn (SET), which is defined as the energy required to overcome drag per unit mass per turn, is a simple concept for evaluating the turn effectiveness of an aircraft. The characteristics of SET vs turn-rate show up the effects of drag polar and wing loading under the given conditions of flight. The simplicity of constant thrust/weight curves and constant $C_{L\max}$ curves in the SET-turn-rate plane permits a convenient separation of their effects on the achievable peak sustained and transient turn-rates. SET is shown to be an absolute minimum at a certain optimum turn-rate for the given configuration, wing loading, and flight conditions. The optimum turn-rate is generally larger than that needed for maximum lift/drag. Analytical expressions of optimum turn-rate, minimum SET, and an index of flatness of the SET-turn-rate curve are obtained on the basis of a recent representation of the drag polar, which seems to be an acceptable approximation over a large range of angles of attack. An example is given to indicate possible applications to parametric studies.

Nomenclature

A	$= 1+2\alpha\cos^2\theta$
B	$= 3(4\alpha\beta-1)$
C	$= \beta+\cos^2\theta$
C_D, C_L, C_T, C_W	$= (D,L,T,W)/Sq$
C_{D0}	= constant term in Eq. (7) (zero-lift drag)
C_1-C_5	= coefficients defined by Eq. (9)
D	= drag
M	= Mach number
S	= wing area
T	= thrust during turn, also absolute temperature in Sec. V
V	= velocity of the aircraft
W	= weight during turn
a	= coefficient of C_L^4 term in Eq. (7)
e	= specific energy of turn (SET)
$e_{\min}$	= minimum SET under specified conditions
g	= gravitational constant
h	= altitude of aircraft
k	= coefficient of C_L^2 term in Eq. (7)
n	= ratio of normal acceleration to g $= V\omega/g$
n'	= load factor (L/W)
p	= air pressure
q	$= \frac{1}{2}\rho V^2$
t	= time
$\Delta\omega$	$= \omega-\omega_{\rm opt}$
α	= angle of attack; also aC_W^2/k
β	$= C_{D0}/kC_W^2$
γ	= ratio of specific heats
ϵ	$= (e-e_{\min})/e_{\min}$
θ	= angle of the aircraft velocity with the horizontal plane
ρ	= air density
ϕ	= angle of the normal to the aircraft trajectory with the horizontal plane
ψ	= angle between the lift and the normal
ω	= angular velocity
$\omega_{\rm opt}$	$= \omega$ at which e is minimum for specified conditions

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I. Introduction

AIRCRAFT maneuverability is frequently discussed in terms of maximum sustained and instantaneous turn-rates and specific excess power. This paper shows that further insight can be obtained by examining the energy requirements of a turning aircraft.

A convenient aerodynamic index of the energy cost of turning an aircraft is the energy required to overcome its air resistance per unit mass per turn, and it is termed specific energy of turn (SET).† This index and the engine characteristics determine the fuel requirements. Evidently, reduction of SET at the design stage could lead to an increase in the range, or a reduction in the aircraft weight.

The plan of this paper is to develop in Sec. II the analytical basis for SET and then to discuss in Sec. III a suitable representation of the drag polar. Analytical relations for minimum SET are subsequently obtained in Sec. IV, and turn performance is then discussed in Sec. V in terms of SET-turn-rate characteristics. An illustrative example is given in Sec. VI to show how the characteristics can be conveniently used in a parametric study. Finally, design implications are discussed in Sec. VII.

II. Specific Energy of Turn

An appropriate form of the equations of motion of the general curvilinear motion of an aircraft is

$$(W/g)\,\mathrm{d}V/\mathrm{d}t = T\cos\alpha - D - W\sin\theta \quad \text{(tangential)} \qquad (1a)$$

$$WV\omega/g = (L+T\sin\alpha)\cos\psi - W\cos\theta\sin\phi \quad \text{(normal)} \qquad (1b)$$

$$(L+T\sin\alpha)\sin\psi - W\cos\theta\cos\phi = 0 \quad \text{(binormal)} \qquad (1c)$$

Here θ and ϕ are the angles made by the aircraft velocity and the normal to the aircraft trajectory with the horizontal plane. It is assumed that the thrust is in the plane of the aircraft velocity and the lift, and is inclined at the angle of attack α with the velocity. Consequently, we are dealing with a

†Since larger mass implies larger lift and hence higher energy requirements, other factors remaining unaltered, specific energy of turn enables one to compare aircrafts of different weight classes. An alternate index could be the angle by which a unit mass can be turned by spending a unit of energy, which would be analogous to specific range. However, its analytical expression becomes slightly more complicated.

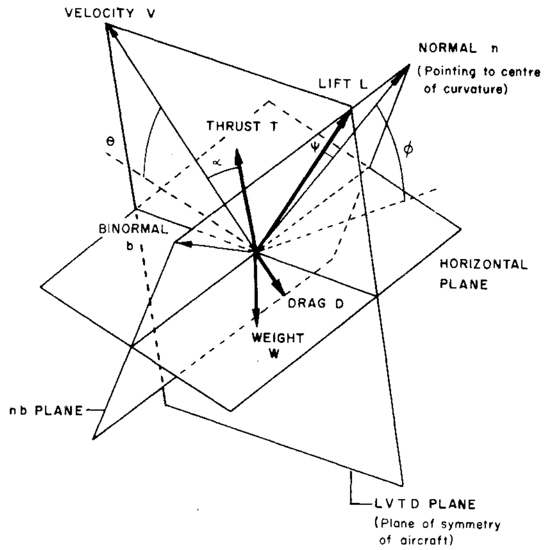

Fig. 1 Sketch with notation.

symmetrical aircraft operating at zero sideslip and zero side force and without thrust vectoring. The lift is inclined at an angle ψ with the normal (Fig. 1).

The energy equation follows from Eq. (1a).

$$DV = TV\cos\alpha - \frac{d}{dt}\left[\tfrac{1}{2}(W/g)V^2 + Wh\right] \qquad (2)$$

The work required to overcome drag is thus provided by the propulsive work or by the loss of the stored-up kinetic or potential energy of the aircraft. It is related to the lift required for centripetal acceleration and for balancing gravity by the following relation, which is obtained from Eqs. (1b) and (1c).

$$(C_L + C_T\sin\alpha)^2 = C_W^2(n^2 + 2n\cos\theta\sin\phi + \cos^2\theta) \qquad (3a)$$

where $n(=V\omega/g)$ is proportional to the normal acceleration.‡

When the angle of attack is small, as in a gentle turn, the term $\sin\alpha$ is small and the left side can be well approximated by C_L^2. When the aircraft is taking a tight turn, α can be large. However, under these conditions, C_L is several times C_W and hence it is much larger than C_T, as the maximum thrust is roughly of the order of the weight. Hence the left side can be approximated by C_L^2 over the entire range of interest.

$$C_L^2 \simeq C_W^2(n^2 + 2n\cos\theta\sin\phi + \cos^2\theta) \qquad (3b)$$

The definition of SET gives the following relation.

$$e = (DV)(g/W)(2\pi/\omega) = 2\pi V^2 C_D / C_W n \qquad (4)$$

If the time scale of the changes in aircraft motion is much larger than the time scale of the relevant unsteady aerodynamic phenomena, C_D can be related to C_L and Mach number by the steady-state drag polar.§

$$C_D = C_D(C_L, M) \qquad (5)$$

Equations (3b-5) can now be used to evaluate e for given n, θ, ϕ, C_W, V, and M. It may be noted that these relations do not impose any restriction on the aircraft trajectory, or on the changes of speed, except that the time scale of these changes be large compared to that of the unsteady aerodynamic phenomena.

‡n is evidently related to the load factor n' by the relation $n'^2 = n^2 + 2n\cos\theta\sin\phi + \cos^2\theta$, which can be seen from Eq. (3b).

§Here we are not explicitly showing dependence on Reynolds number.

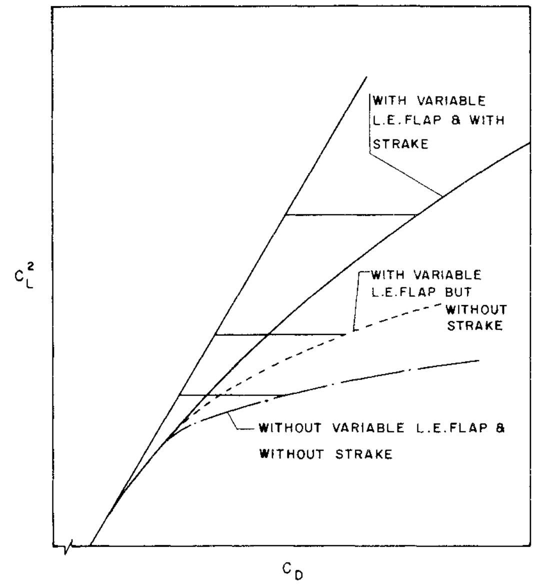

Fig. 2 Departure from the parabolic drag polar (for YF-16, based on Fig. 23 of Ref. 2).

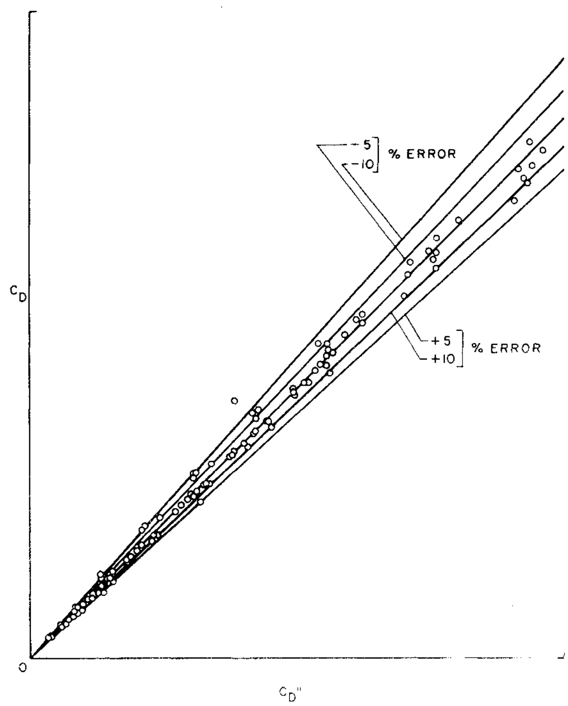

Fig. 3 Comparison of experimental values C_D with those values C_D'' obtained from the representation[1] given by Eq. (7).

III. Drag Polar Representation

The well-known representation of the drag polar

$$C_D = C_{D0} + kC_L^2 \qquad (6)$$

is a very useful approximation for relatively small angles of attack encountered in cruise and climb. However, the range of angle of attack in maneuvers is so large that the above representation ceases to be a good approximation.

Figure 2 shows the drag polar[2] of a typical configuration used in the development of F-16 in C_D vs C_L^2 coordinates to indicate increasing departures from Eq. (6) with increasing C_L^2. A recent representation[1] generalizes Eq. (6) by adding a correction term which is insignificant at small angles of attack but which makes an important contribution at large angles of attack. Figure 3 compares the C_D values given by the following representation,[1] in which C_{D0}, k, and a were determined by a least-square fit for NAL data on an aircraft configuration in the Mach number range of 0.5-1.5, with the corresponding experimental values.

$$C_D = C_{D0} + kC_L^2 + aC_L^4 \tag{7}$$

The above representation¶ is used hereafter.

IV. Minimum Set

Substitution of Eqs. (3b) and (7) in Eq. (r) yields

$$e/2\pi V^2 = kC_W(C_1/n + C_2 + C_3 n + C_4 n^2 + C_5 n^3) \tag{8}$$

where the coefficients C_1 to C_5 are given** by

$$C_1 = \beta + \cos^2\theta + \alpha\cos^4\theta$$

$$C_2 = 2\cos\theta\sin\phi(1 + 2\alpha\cos^2\theta)$$

$$C_3 = 1 + 2\alpha\cos^2\theta + 4\alpha\cos^2\theta\sin^2\phi$$

$$C_4 = 4\alpha\cos\theta\sin\phi$$

$$C_5 = \alpha$$

$$\alpha = aC_W^2/k \quad \beta = C_{D0}/kC_w^2 \tag{9}$$

Evidently, for given θ and ϕ, the $e/2\pi V^2$vs n curve is determined by the three nondimensional parameters, namely, kC_w, α, and β. If KC_w is increased, keeping α and β constant, the scale of $e/2\pi V^2$ increases linearly. The extremum value of e occurs when $(\partial e/\partial n)$ is zero, that is, when

$$3C_5 n^4 + 2C_4 n^3 + C_3 n^2 - C_1 = 0 \tag{10}$$

We consider the simpler case of an aircraft turning in such a way that the normal is horizontal. ϕ is zero at the given instant. However, the aircraft may move in a helical path as θ is not required to be zero. Then Eqs. (8-10) simplify as C_2 and C_4 are zero. The turn-rate ω_{opt} satisfying Eq. (10) is then given†† by

$$\omega_{opt} = (g/V)[\{(4A^2+B)^{1/2} - A\}/6\alpha]^{1/2} \tag{11}$$

where

$$A = 1 + 2\alpha\cos^2\theta \qquad B = 3(4\alpha\beta - 1) \tag{12}$$

The above rate is optimum from the point of view of energy requirements, as the corresponding energy requirement is an absolute minimum. The minimum value of SET is given by

$$e_{min}/2\pi V^2 = kC_w(2/27\alpha)^{1/2}\frac{[A(4A^2+B)^{1/2} + 2A^2 + B]}{[(4A^2+B)^{1/2} - A]^{1/2}} \tag{13}$$

¶Some obvious variations like replacing C_L by $(C_L - C_{L0})$ and aC_L^4 by aC_L^n are possible. Close examination[1] of data reveals that the approximation provided by Eq. (7) is acceptable considering uncertainties in corrections for support and wall interference and Reynolds number changes, when the angle of attack is large.

**Note α is used for angle of attack as well as the parameter defined by Eq. (9), as there is no likelihood of confusion.

††The roots of Eq. (10) (for $\phi=0$) are $n^2 = \{\pm(4A^2+B)^{1/2} - A\}/6\alpha$. The negative sign is inadmissible as $3A^2+B$ is positive for a, $k>0$. Hence, the root given by Eq. (11) is the only relevant root of Eq. (10).

It is to be noted that the turn-rate for max. (L/D) for a horizontal turn (i.e., $\theta=\phi=0$) is given by $\omega = (g/V)\ [\{(1+12C_{D0}^2/k^2)^{1/2} - 1 - 6\alpha\}/6\alpha]^{1/2}$, and it is smaller than ω_{opt}. However, the difference approaches zero as $\beta \to \infty$.

Also, when the aircraft is turning in a horizontal plane, θ and ϕ are zero and the second derivative of the SET-turn-rate characteristic is given by

$$(\partial^2 e/\partial n^2)/2\pi V^2 = (kC_w/6\alpha n^3)[(3A^2+B) + 36\alpha^2 n^4] \tag{14a}$$

The above relation takes the following form at $\omega = \omega_{opt}$.

$$(\partial^2 e/\partial n^2)/2\pi V^2 = (kC_w/3\alpha n^3)(4A^2+B)^{1/2} \times [(4A^2+B)^{1/2} - A] \tag{14b}$$

Clearly, the second derivative is positive for all $n>0$, as $(3A^2+B)$ is positive for a, $k>0$. Hence it follows that SET has an absolute minimum at $\omega = \omega_{opt}$ for a, $k>0$.

It is of some interest to know how flat the SET-turn-rate curve is near the minima. Suppose ω is $\omega_{opt} + \Delta\omega$, and the corresponding value of SET is $(1+\epsilon)e_{min}$. Then, for sufficiently small $\Delta\omega$, $(1+\epsilon)e_{min} \simeq e_{min} + \frac{1}{2}\partial^2 e/\partial n^2 (V\Delta\omega/g)^2$. Consequently, $\Delta\omega$ is given by

$$\Delta\omega = \pm(g/V)[\tfrac{1}{3}(\epsilon/\alpha)^{1/2}\{A(4A^2+B)^{1/2} + 2A^2 + B\}^{1/2}/(4A^2+B)^{1/4}] \tag{15}$$

The above expression can be used for estimating the permissible departure of ω from ω_{opt}, if SET is allowed to exceed the minimum by ϵe_{min}. A large value of $\Delta\omega$ for a given ϵ indicates a relatively flat curve.

Approximate forms of Eqs. (11-15) are given in the Appendix. The following approximate formulas are particularly suitable for quick estimates, when $\alpha\beta \ll 1$ and $\beta \gg 1$.

$$\omega_{opt} = (g/V)\beta^{1/2} \tag{16}$$

$$e_{min}/2\pi V^2 = 2kC_w\beta^{1/2} \tag{17}$$

$$\Delta\omega = \pm(g/V)(2\epsilon\beta)^{1/2} \tag{18}$$

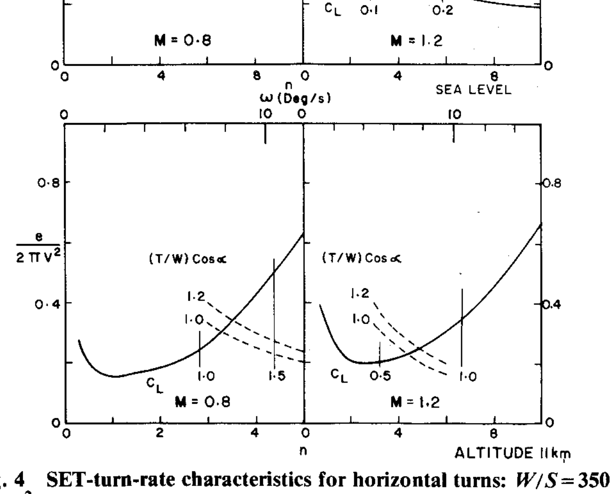

Fig. 4 SET-turn-rate characteristics for horizontal turns: $W/S = 350$ kg/m², $C_{D0} = 0.015$, $k = 0.15$, $a = 0.075$, $\theta = \phi = 0$; (----) constant $(T/W)\cos\alpha$ curves. Constant C_L curves are vertical lines. ISA and ISA + 30° conditions are indicated by corresponding scales of ω.

Table 1 Typical values of ω_{opt}, e_{min}, and $\Delta\omega$ for horizontal turns ($\theta=\phi=0$)[a]

		Equation	Sea-level		11 km altitude	
M			0.8	1.2	0.8	1.2
ω_{opt}	exact	(11)	8.307	16.28	2.839	4.410
	approx.	(16)	8.642	18.33	2.221	4.723
	approx.	(A1a)	8.168	15.41	2.589	4.093
$\frac{e_{min}}{2\pi V^2}$	exact	(13)	.1001	.1870	.1538	.1993
	approx.	(17)	.0949	.1789	.0949	.1789
	approx.	(A1b)	.1004	.1891	.1579	.2015
$\Delta\omega$	exact (for $\epsilon=0.1$)	(15)	3.437	6.521	1.134	1.736
	approx.	(18)	3.865	.8199	0.998	2.117
	approx.	(A1c)	3.555	6.455	1.127	1.723

[a] $W/S=350$ Kg/m^2; C_{D0}, k, a are 0.015, 0.15, and 0.075, respectively, at $M=0.8$, and 0.04, 0.2, and 0.1, respectively, at $M=1.2$; ISA conditions

Table 2 Typical values of α and β for horizontal turns[a]

	Sea-level		11 km altitude	
M	0.8	1.2	0.8	1.2
α	.0028	.0006	.0565	.0112
β	17.56	177.8	.8850	8.961

[a] W/S, k, C_{D0}, and a as in Table 1.

The errors arising from the above approximate relations are generally of the order of 10% as indicated in Table 1. (Errors are larger for high-altitude subsonic performance.) Improved approximations are given in the Appendix by Eq. (A1).

V. SET-Turn-Rate Characteristics

Figure 4 shows typical SET-turn-rate characteristics at sea level and high altitude for high subsonic and low supersonic performance for horizontal turns, that is for $\theta=\phi=0$. The curves are for a wing loading of 350 kg/m^2 and for C_{D0}, k, and a of 0.015, 0.15, and 0.075, respectively, at $M=0.8$; and 0.04, 0.2, and 0.1, respectively, at $M=1.2$. The abscissa gives the normal acceleration ($n=V\omega/g$) and also the turn-rate ω.

It is seen that ω_{opt} is the lowest for the high-altitude subsonic case at about 2.8 deg/sec. It is higher at about 4.4 and 8.3 deg/sec for the high-altitude supersonic and sea-level subsonic case. The normal acceleration associated with ω_{opt} at sea level for $M=1.2$ is much too large to be of any practical interest. (This variation arises mainly because of large variation in β—Eq. (16) and Table 2.) $e_{min}/2\pi V^2$ is found to lie in a rather narrow range of 0.1 to 0.2, the lowest being for the sea-level subsonic case.

In all four cases shown in Fig. 4, the characteristic drops rapidly as ω increases to ω_{opt}. As ω increases beyond ω_{opt}, the curve rises relatively slowly. The values of acceleration and turn-rate in the second range are of primary interest.

Effect of Thrust/Weight Ratio

Thrust/weight ratio‡‡ can be easily calculated from Eqs. (2, 4, and 8) for constant-speed horizontal turns.

$$(T/W)\cos\alpha=(e/2\pi V^2)n=kC_w(C_1+C_3n^2+C_5n^4) \quad (19)$$

The first relation shows that the curves of constant (T/W) $\cos\alpha$ are hyperbolas in the SET-turn-rate plane. Changes in the maximum sustained turn-rate arising from changes in (T/W) can be readily seen from Fig. 4 if it is assumed that $\cos\alpha$ remains approximately unchanged. The largest improvement in sustained turn-rates is found in the subsonic sea-level turns when $(T/W)\cos\alpha$ is changed from 1 to 1.2.

‡‡Evidently, the values of thrust and weight correspond to the instant for which the calculation is made.

Effect of $C_{L\max}$

C_L is determined by n according to Eq. (3b) for a given W/S and atmospheric conditions. Constant C_L curves are consequently vertical straight lines in the SET-turn-rate plane. The effect of an increase in $C_{L\max}$ on the maximum instantaneous turn-rate can be estimated from the constant C_L lines in the SET-turn-rate plane (Fig. 4).

Effect of Tropical Atmosphere

Effects of departures from ISA conditions are also easy to see. A change from ISA to, say, ISA+30° does not affect $C_w[=W/\{S(\frac{1}{2}\gamma pM^2)\}]$, as pressure and Mach number are held constant. Also, the values of C_1, C_3, and C_5 are not altered. Hence the only modification that is required is to change the scale of ω according to $n=(M\sqrt{\gamma RT/g})$, as shown in Fig. 4.

VI. An Illustrative Example

The characteristics given in Fig. 4 can be regarded as those of a baseline aerodynamic design. One can now easily calculate the changes in the SET-turn-rate characteristic due to 1) a 20% reduction in W/S from 350 to 280 kg/m^2, 2) a 10% reduction in C_{D0} (from 0.015 to 0.0135 in the subsonic range and from 0.04 to 0.036 in the supersonic range), 3) a 10% reduction in k (from 0.15 to 0.135 in the subsonic range and from 0.2 to 0.18 in the supersonic range), and 4) a 10% reduction in a (from 0.075 to 0.0675 in the subsonic range and from 0.1 to 0.09 in the supersonic range), keeping other quantities fixed (Figs. 5 and 6).

Effect of Wing Loading

A reduction in wing loading results in a substantial saving of energy at high altitude at $M=0.8$ and 1.2 and also at sea level at $M=0.8$. The hyperbolic character of constant (T/W) $\cos\alpha$ curves yields considerable increase in peak sustained turn-rate. For instance, it increases from about 7.5 to about 8.8 deg/sec at high altitude for $(T/W)\cos\alpha=1$. The energy requirement, however, increases in the sea-level supersonic operation which is of marginal importance. It can be noted that the reduction in W/S increases ω_{opt} and makes

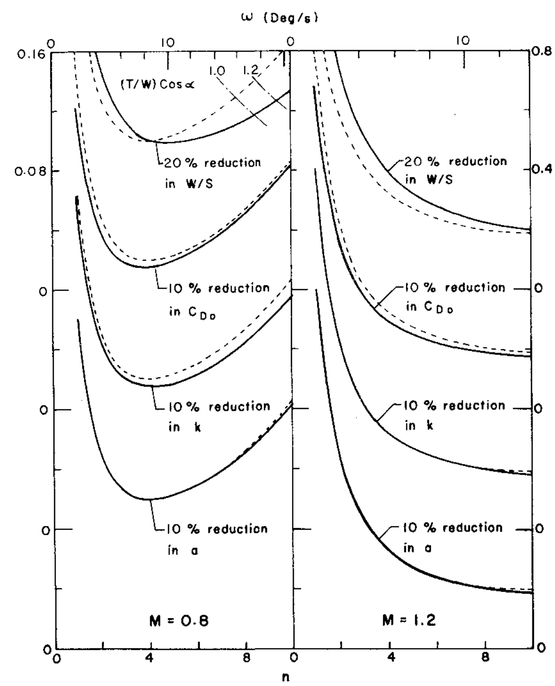

Fig. 5 Sensitivity of SET-turn-rate characteristics for sea-level horizontal turns, (----) baseline characteristics for conditions indicated in Fig. 4.

the curve flatter, which can be immediately inferred from Eqs. (16) and (18), since β increases as W/S is reduced. However, $e_{\min}/2\pi V^2$ hardly changes, which can also be seen from Eq. (17).

Evidently, reduction in wing loading is one of the most effective ways of reducing the energy requirements in a wide range of turns of practical interest. As the characteristic becomes flatter, the penalty for not operating at $\omega_{\rm opt}$ markedly diminishes.

Effect of C_{D0}

The effect of a reduction in C_{D0} is seen to be most noticeable at sea level. The energy requirements are reduced, but the reduction falls with increasing turn-rate. Subsonic high-altitude performance is negligibly affected. Approximate relations (16-18) suggest reduction in $\omega_{\rm opt}$, $e_{\min}$, and $\Delta\omega$.

Effect of k

Effects of a reduction in k are most pronounced in subsonic sea-level turns, and increase at larger turn-rates. Supersonic turn performance is not appreciably affected. Approximate relations (16-18) suggest increase in $\omega_{\rm opt}$ and $\Delta\omega$ and decrease of $e_{\min}$.

Effect of a

Changes in a affect the characteristic noticeably when C_L is large, i.e., at large turn-rates and at high altitude.

VII. Comments on Design Implications

Design for greater maneuverability can be considered not only in terms of peak sustained and instantaneous turn-rates, but also in terms of the SET-turn-rate characteristics. For instance, Figs. 4 and 6 suggest that if we compare the benefits in high-altitude turns at $M=0.8$ of increasing $(T/W)\cos\alpha$ from 1 to 1.2 with those of decreasing W/S from 350 to 280 kg/m^2, the latter are more attractive not only because the gain in sustained turn rate is larger, but also because the energy requirements are significantly brought down over a range of turn rates.

Equations (16-18) give a quick indication of the influence of important parameters. Evidently, one would generally want to increase $\omega_{\rm opt}$ and $\Delta\omega$ and hence large values of β are desirable. Since q depends on flight conditions, $(C_{D0}/k)^{1/2}/(W/S)$ can be taken as a figure of merit. Similarly, $e_{\min}$ can be reduced by decreasing (kC_{D0}).

Appendix: Approximations for SET-Turn-Rate Characteristics

Table 2 gives values of α and β for the case considered in Sec. V. Evidently, one useful approximation is $\alpha \ll 1$, or formally $\alpha \to 0$.

Approximation I $(\alpha \ll 1, \phi = 0)$

We expand various expressions in powers of α and retain the lowest two orders. For most purposes, the lowest order terms would suffice. Equations (11, 13, and 15) give

$$\omega_{\rm opt} = (g/V)C^{1/2}[1-(\alpha/2C)(3C^2+2\cos^2\theta-\cos^4\theta)] \quad \text{(A1a)}$$

$$e_{\min}/2\pi V^2 = 2kC_w C^{1/2}[1+(\alpha/2C)\times(C^2+2C\cos^2\theta+2\cos^4\theta)] \quad \text{(A1b)}$$

$$\Delta\omega = \pm(g/V)(2\epsilon C)^{1/2}[1-(\alpha/2C)\times(4C^2+C\cos^2\theta-\cos^4\theta)] \quad \text{(A1c)}$$

where C is $\beta+\cos^2\theta$. The terms of order α in the above equations are seen to be negligible when $\alpha \ll 0.4$, if $\beta \ll 1$, or when $\alpha \ll 2C/(4C^2+C-1)$, if $\beta \sim 1$, or when $\alpha \ll 1/2\beta$, if $\beta \gg 1$ (Table 2). When this condition is satisfied, the above approximations further simplify to

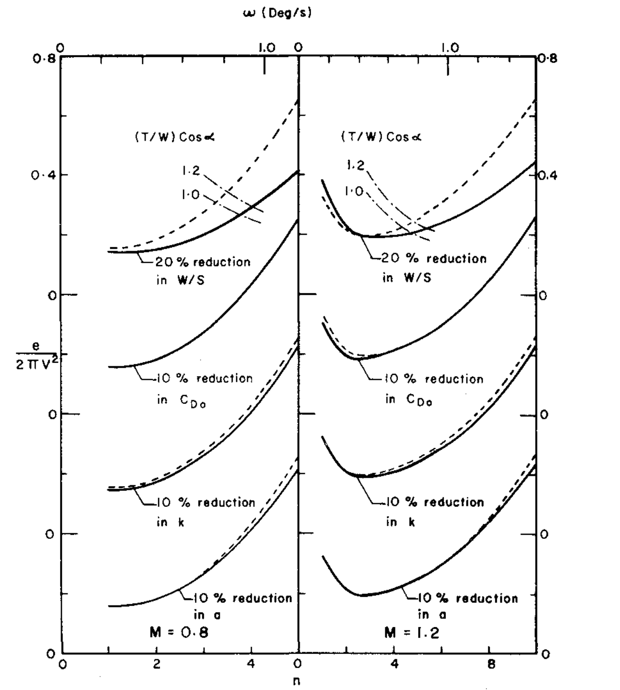

Fig. 6 Sensitivity of SET-turn-rate characteristics for high altitude horizontal turns, (----) baseline characteristics for conditions indicated in Fig. 4.

$$\omega_{\rm opt} = (g/V)C^{1/2} \quad \text{(A2a)}$$

$$e_{\min}/2\pi V^2 = 2kC_w C^{1/2} \quad \text{(A2b)}$$

$$\Delta\omega = \pm(g/V)(2\epsilon C)^{1/2} \quad \text{(A2c)}$$

It is interesting to note that if the classical drag polar representation [Eq. (6)] holds, a and hence α can be taken to be zero in Eq. (9) and the SET-turn-rate characteristic Eq. (8) becomes

$$e/2\pi V^2 = kC_w(C/n+n) \quad \text{(A3)}$$

for $\phi=0$.

Equation (A2) is then exact, as can be readily seen. Also the turn-rate which maximizes (L/D) is given by

$$\omega = (g/V)(C-2\cos^2\theta)^{1/2} \quad \text{(A4)}$$

and hence it is generally smaller than $\omega_{\rm opt}$. So the difference between $\omega_{\rm opt}$ and ω for max (L/D) is not critically dependent on the departure from the classical approximation to the drag polar.

Incidentally, Eq. (A3) begins to deviate from the exact [Eq. (8)] for small α when n begins to be comparable to $(C/\alpha)^{1/2}$. For instance, with $C_w=0.1$, and $(C_{D0}/a)^{1/4}=0.5$, the nonlinearity in the C_D vs C_L^2 curve has significant effects for normal accelerations greater than $5g$.

Approximation II $(\alpha\beta \ll 1, \beta \gg 1, \phi=0)$

$\alpha\beta(=aC_{D0}/k^2)$ is typically of the order of 0.1. Furthermore, if the high-altitude subsonic case is excluded, $\beta \gg 1$. Under these conditions, Eqs. (11, 13, and 15) simplify to Eqs. (16-18). The implications of these simple relations have been discussed earlier. One additional point to be noted is that the flight direction (i.e., θ) does not influence the energy requirements to the order of this approximation.

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